

Multi-satellite SLR analysis including LARES/LARES-2 SLR data

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Abstract

In this study, the contributions of LARES and LARES-2 SLR data to multi-satellite SLR analyses in combination with LAGEOS-1/2 are validated. Since, compared to LAGEOS-1/2, LARES orbits the Earth on a lower altitude (1400 km) it is more sensitive to the time-variable Earth's gravity field and therefore enables to co-estimate low-degree gravity field coefficients. However, for this purpose, the orbit parametrizations of the satellites have to be adapted.

Further, the spherical satellite LARES-2 was launched at July 13, 2022 and the first SLR measurements are available. The impact of the first few months of LARES-2 data on multi-satellite SLR analyses is studied by comparing the estimated parameters, e.g., the Earth rotation parameters and station coordinates, with external quality metrics. The first results confirm the high quality of the LARES-2 SLR observations.

1. Introduction

Even if nowadays the time-variable Earth's gravity field is mainly determined by dedicated gravimetry satellite missions, i.e., the Gravity Recovery And Climate Experiment (GRACE) (Tapley et al. 2004) and GRACE Follow-on (Landerer et al. 2020), the Satellite Laser Ranging (SLR) technique is still essential to determine reliable low-degree gravity field coefficients, especially the zonal spherical harmonic coefficients C_{20} and C_{30} (e.g., Bianco et al. (1998), Loomis et al. (2020)). Since LARES is orbiting the Earth at an altitude of 1400 km it has an increased sensitivity to the Earth's gravity field and therefore it enables to the co-estimation of low-degree Earth's gravity field coefficients in the parameter adjustment (e.g., Bloßfeld et al. (2015), Bloßfeld et al. (2018)). Combining LARES with LAGEOS-1/2 SLR data on the normal equation level using Variance Component Estimation (VCE), the suitable orbit parametrizations for LARES are studied (see Section 2).

Furthermore, multi-satellite SLR analyses including LARES-2 data are validated (see Section 3). Due to the very small area-to-mass ratio and an orbit altitude of 6000 km, the non-gravitational perturbing forces are minimized and orbit modeling is therefore simplified.

The SLR analyses are performed with the Bernese GNSS Software (Dach et al. 2015), where the 7-day satellite orbits are represented by six initial osculating orbital elements referring to the beginning of the arc and five dynamic orbit parameters, i.e., one constant acceleration in along-track (S_0) and Once-Per-Revolution (OPR) sine and cosine accelerations in along-track (S_S, S_C) and cross-track (C_S, C_C). To partially absorb possible air drag modeling deficiency for LARES, daily pseudo-stochastic pulses (instantaneous velocity changes) in along-track are set up. With the background mod-

Table 1: Background models.

Models	Description
Reference frame	SLRF2014 ¹
ERPs	IERS-14-C04 (Bizouard et al. 2019)
Nutation model	IAU2000 (Mathews, Herring, and Buffett 2002)
Subdaily pole model	DESAI (Desai and Sibois 2016)
Ocean tide model	FES2014b: d/o 30 (Lyard et al. 2021) + admittances
Earth tides	Solid Earth tides, pole tides and ocean pole tides: IERS 2010 (Petit and Luzum 2010)
Loading corrections	Ocean tidal loading: FES2014 Atmospheric tidal loading: Ray and Ponte (Ray and Ponte 2003)
De-aliasing products	Atmosphere + Ocean RL06: d/o 30 incl. S1- and S2-atmosphere tides (Dobslaw et al. 2017)
Earth gravity field	GGM05S: d/o 90 (Ries et al. 2018)

els listed in Table 1, the orbital parameters are estimated simultaneously with geodetic and instrument parameters (see Table 2). The Earth Rotation Parameters (ERPs) are modeled by piecewise linear polygons with daily polygon vertices estimated at 0^h epoch (Thaller et al. 2012). In addition, the 4th offset of UT1-UTC is fixed to the IERS-14-C04 a priori series. For the comparison of the estimated ERP values with the reference series, the estimated polygon is evaluated at the 12^h epochs (Thaller et al. 2012).

In case the parametrization is adapted, it is explicitly mentioned in the corresponding section.

2. Impact of SLR LARES data on co-estimated Earth geopotential coefficients

To study the impact of LARES data on the co-estimation of fully normalized Earth geopotential coefficients, the parameter space of the multi-satellite SLR analyses, based on LAGEOS-1/2 and LARES data within the years 2015-2020, is extended by the Earth's gravity field coefficients from degree/order (d/o) 2 up to 4. Since the geocenter coordinates are estimated as translation vectors, the coefficients of d/o 1 are not estimated. Due to strong correlations between the OPR cosine accelerations in cross-track (C_C) and the Earth's gravity field coefficient C_{20} (e.g., Jäggi et al. (2012) or Bloßfeld et al. (2014)), as well as between the OPR sine accelerations in along-track (S_S) and C_{30} (e.g., Bloßfeld et al. (2018)), the orbital parameterization introduced in Section 1 has to be adapted (Geisser et al. 2023).

Table 2: Estimated parameters.

Parametrization/Satellites	LAGEOS-1/2	LARES	LARES-2
Osculating elements	$a, e, i, \Omega, \omega, u_0$ (1 set per 7 days)		
Constant and once-per-revolution accelerations	S_0, S_S, S_C, C_S, C_C (1 set per 7 days)		
Pseudo-stochastic pulses	no pulses	in along-track (twice per day)	no pulses
Earth Rotation Parameters	X-Pole, Y-pole, UT1-UTC (1 set per 1 day)		
Geocenter coordinates	1 set per 7 days		
Station coordinates	NNR and NNT minimum constraint (1 set per 7 days)		
Range biases	selected stations	1 set per 7 days for all stations	all stations

¹ ftp://edc.dgfi.tum.de/pub/slr/aux_data/ILRS_Data_Handling_File.snx

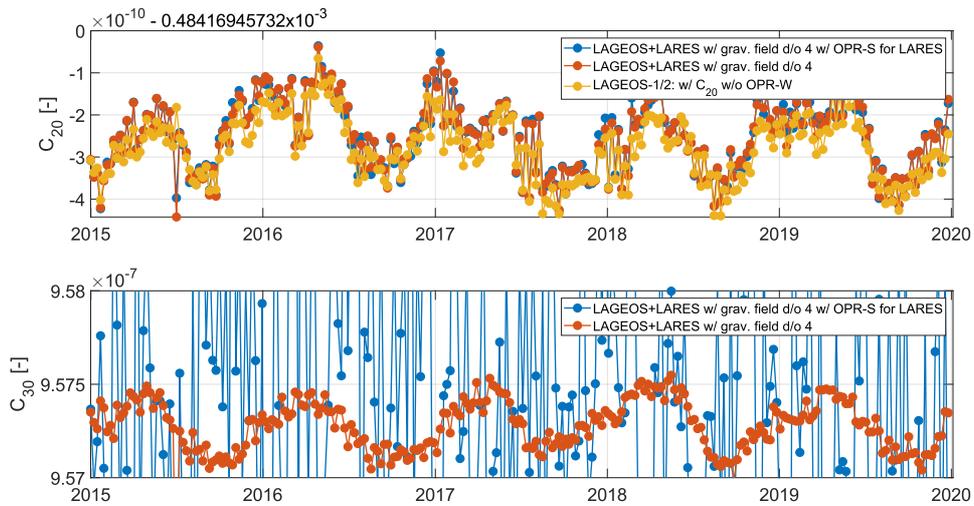


Figure 1: Estimated C_{20} for LAGEOS-1/2 analyses and with the inclusion of LARES with or without OPR-S accelerations for LARES (top). Co-estimated C_{30} for different SLR solutions (bottom).

Hence, the OPR accelerations in C are omitted for LAGEOS-1/2 and LARES, while the OPR accelerations in S are only neglected for LARES. Consequently, the inclusion of LARES does not degrade the high quality of C_{20} estimates from a LAGEOS-1/2 SLR analysis (see Figure 1, top). In addition, the omission of OPR accelerations in S enables to also co-estimate a reliable C_{30} (see Figure 1, bottom). For further information see Geisser et al. (2023).

3. Contribution of LARES-2 data to SLR analyses

In this section, the contribution of LARES-2 SLR data is studied by comparing weekly SLR analyses based on LAGEOS-1/2, LARES and LARES-2. Since LARES-2 was launched on July 13, 2022, weekly SLR analyses are generated starting from mid of July 2022 to mid of November 2022. Already in the beginning of the tracking campaign of LARES-2, the number of Normal Points (NPs) reached almost the same amount as for LAGEOS-2 (see Figure 2).

For these analyses, the a priori Earth’s gravity field model GGM05S (listed in Table 1) is replaced by a time-variable gravity field model, a so-called fitted signal model (FSM), provided by the Combination Service for Time-variable Gravity Fields (COST-G) (Peter et al. 2022).

Figure 3 shows the weekly estimated weights for LARES and LARES-2 given by VCE w.r.t. LAGEOS-1/2. LARES-2 gets on average a 3.24 times higher weight than

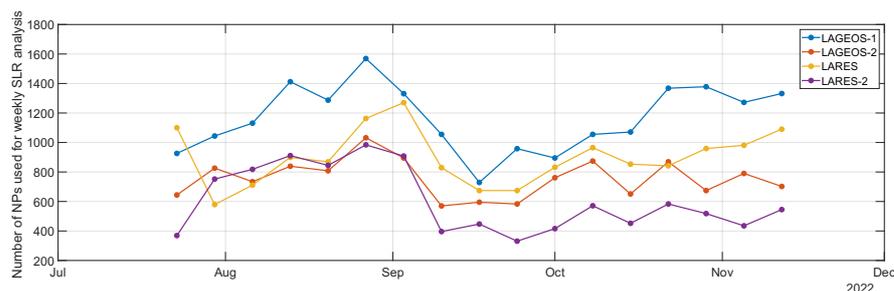


Figure 2: Number of NPs used to perform weekly SLR analyses.

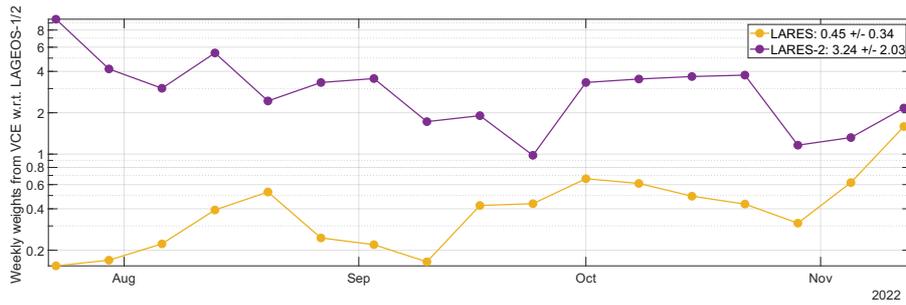


Figure 3: Weekly weights for LARES and LARES-2 w.r.t. LAGEOS-1/2 as derived by the VCE.

LAGEOS-1/2. However, LARES is only weighted half as LAGEOS-1/2. Hence, VCE indicates that the LARES-2 observations fit better to the mathematical model than the other satellites.

The quality of the multi-satellite SLR analyses are validated by comparing geodetic parameters, i.e., Earth Rotation Parameters (ERPs) and station coordinates, with reference solutions.

If LARES-2 data are included, the bias of the Y-pole is more than doubled. Nevertheless, the Weighted Root Mean Square (WRMS, weighted with the formal error) of the Y-pole can be reduced by 14 μs (see Table 3). Furthermore, the increased WRMS of UT1-UTC can be explained by the unbalanced inclinations of the satellite orbits, i.e., prograde (LAGEOS-1) or retrograde (LAGEOS-2, LARES, LARES-2) motion. The high weight of LARES-2 shifts the estimated ascending node systematically and due to the correlation with UT1-UTC, it also affects the estimation of UT1-UTC. This phenomenon was confirmed by an experiment of LAGEOS-1/2 SLR analyses, where different weights were assigned to LAGEOS-1 and -2 to enforce an imbalance.

Table 3: Estimated ERPs are compared w.r.t. the IERS-14-C04 reference series.

	X-pole [μs]		Y-pole [μs]		UT1-UTC [μs]	
	Bias	WRMS	Bias	WRMS	Bias	WRMS
LAGEOS-1/2, LARES	152.2	198.5	11.7	154.6	-9.5	69.4
LAGEOS-1/2, LARES, LARES-2	158.2	198.5	32.0	140.6	-5.1	76.0

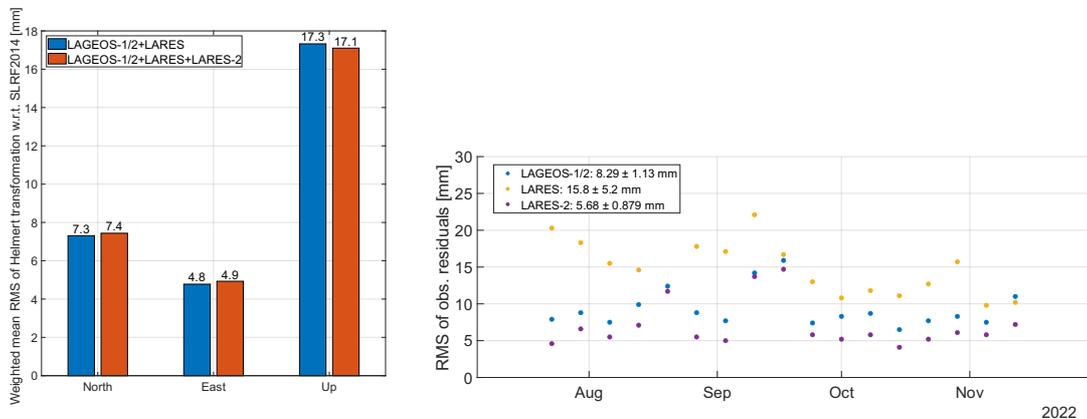


Figure 4: Weighted mean RMS of the Helmert transformation w.r.t. SLRF2014 (left). RMS of observation residuals per satellite group (right).

The weighted mean RMS (weighted with the number of used core stations) of the Helmert transformation w.r.t. SLRF2014 is used to determine the quality of the station coordinates (see Figure 4, left). If LARES-2 is included, the station coordinates are not changing significantly.

Figure 4 (right) shows the RMS of the observation residuals per satellite group, i.e., LAGEOS-1/2, LARES and LARES-2. The mean RMS of the LARES-2 observation residuals are 30 % smaller than for LAGEOS-1/2. Furthermore, LARES has the largest observation residuals, which is to be expected due to the lower altitude and therefore more difficult orbit modeling.

4. Conclusion and Discussion

Dynamic orbit parameters are correlated with the Earth's gravity field coefficients. Therefore, to estimate reliable gravity field coefficients C_{20} and C_{30} within multi-satellite SLR analyses based on data of LAGEOS-1/2 and LARES, the orbit parametrization has to be adapted.

With the launch of LARES-2 in 2022, there is a new target in space, which can be used for SLR analysis. In our analyses, the inclusion of LARES-2 SLR data does slightly degrade the multi-satellite SLR analysis based on LAGEOS-1/2 and LARES. Nevertheless, the mean RMS of the observation residuals of LARES-2 with 5.68 mm confirms the high quality of the SLR observations of LARES-2. The reason that the mean RMS of the observation residuals compared with LAGEOS-1/2 is improved by 30 % can probably be explained by the smaller area-to-mass ratio.

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